Long-range degree correlations reveal structural crossover in critical random graphs

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Long-range degree correlations and fractality are two key concepts characterizing large-scale structure in complex networks, yet their relationship remains largely unclear. Here, we address this problem in random graphs at the percolation critical point, where the incipient percolation cluster is fractal and degree correlations extend throughout the cluster. Starting from the conditional degree distribution of nodes at shortest-path distance l from a degree-k seed node, we derive the expected ball volume, defined as the number of nodes within distance l from the seed. This yields an explicit expression of the ball volume consisting of a seed-degree-dependent term and an asymptotic fractal-growth term associated with the critical percolation cluster. The results reveal a degree-dependent structural crossover: at short distances, the ball volume grows linearly with the product of the seed degree and ball radius, whereas at larger distances it crosses over to the fractal scaling regime. The corresponding crossover length increases systematically with seed degree, giving a quantitative description of how seed degree controls the approach to the global fractal scaling. Our framework shows that long-range degree correlations are not merely closely related to fractality, but describe the degree-dependent crossover from the local growth regime toward the global fractal regime. More broadly, the results establish a concrete link between correlations beyond nearest neighbors and fractal scaling in critical networks, with implications for multiscale transport and diffusion on complex networks.

Keywords: complex networks, percolation criticality, fractality, long-range degree correlations

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