Localization-delocalization transition in physical networks

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ABSTRACT

Physical networks embedded in three-dimensional space under physical constraints are omnipresent in physical and biological systems ranging from self-assembling systems to biological neural networks, to mention a few. Recent studies have revealed that the physicality of networks results in emergent structural features, such as topological entanglement, bundling, and correlations between network topology and physical layout. In particular, correlations $v_i \sim k_i^\alpha$ between node degree k_i and node volume v_i are widely observed in empirical physical networks, and linear correlations ($\alpha = 1$) are shown to emerge in simple random walk models of physical networks.

In this work, we study the impact of degree-volume correlations on the Laplacian dynamics on physical networks. To this end, we systematically investigate the so-called physical Laplacian $\mathbf{Q}_\mathrm{P} = \mathbf{V}^{-1/2}\mathbf{Q}_\mathrm{G}\mathbf{V}^{-1/2}$ with graph Laplacian $\mathbf{Q}_\mathrm{G}$ and volume matrix $\mathbf{V} = \mathrm{diag}(\{v_i\}_i)$, i.e., a volume-weighted graph Laplacian that captures diffusion-like dynamics on nodes with volumes. Our focus lies on the roles of correlation strength α and network topology on the dynamics, and we examine the spectral properties of physical Laplacians in synthetic and empirical networks. Our results demonstrate that the emergent volume-degree correlations in physical networks can suppress or mitigate the effect of high-degree nodes, preventing extreme localizations of eigenvectors in heterogeneous networks. While our motivation originates from physical networks, our results can also apply to network-of-networks and multilayer networks with heterogeneous layer sizes.

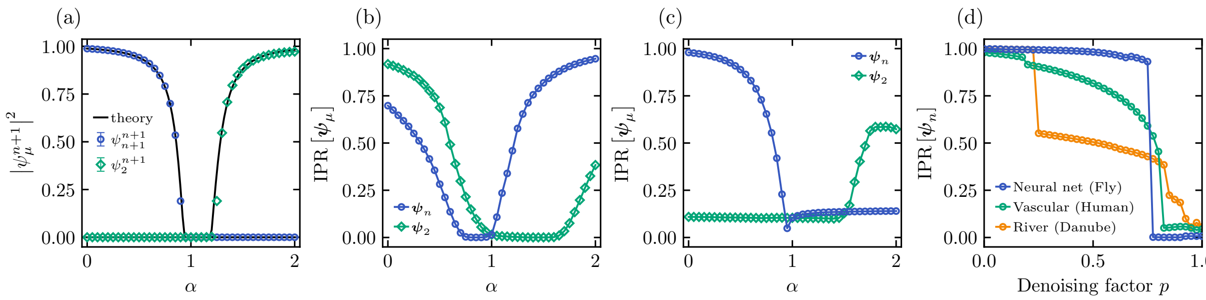

FIGURE 1. (a) The $(n+1)$-th component of leading and Fiedler eigenvectors of the c-regular random graph of n vertices with a randomly attached outlier node of degree d ($n = 10^5$, $c = 10$, $d = 316$). (b–c) The inverse participation ratios (IPRs) of the leading and Fiedler eigenvectors in physical networks with (b) Erdős-Rényi random graph ($n = 10^4$, $\langle k \rangle = 10$) and (c) configuration model with power-law degree distribution $p_k \sim k^{-\gamma}$ ($n = 10^4$, $\gamma = 3$) as combinatorial networks and volumes assigned via $v_i \sim k_i^\alpha$. (d) The IPRs of the leading eigenvector of empirical networks with different denoising factor p, where the volume of each node i is denoised as $\tilde{v}_i = v_i \exp\left[p \log\left\{v_\mathrm{fit}(k_i)/v_i\right\}\right]$.

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