Multifractality of scale-free networks

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Topological configurations of real-world fractal scale-free networks (FSFNs), such as the World-Wide-Web, often exhibit the multifractal property [1]. The multifractal analysis of a complex network enables us to understand how connectivity between nodes are heterogeneous and correlated and provides insight into how structural features of the network influence its functionality [2,3]. We have shown analytically, in our previous work [1], that the (u, v)-flower [4], a typical deterministic model of FSFNs, takes a bifractal structure which is characterized by the two distinct values of the Lifshitz-Hölder exponents. In addition, it has been clarified that any FSFN satisfying a certain condition possesses the bifractal nature. It is, however, still unclear how the two values of the Lifshitz-Hölder exponents are linked to the local structure of the network. Furthermore, we must elucidate whether the bifractal property of deterministic FSFNs can be an origin of the multifractality observed in the real-world networks.

In this work, we examine the multifractal property of a wide class of FSFNs formed by the generalized deterministic model we presented in the previous meeting, in which every edge of the network in the previous generation is replaced by a small graph called a generator. We first prove analytically that any FSFN constructed by this model exhibits the bifractal nature. Next, we discuss to which parts of the FSFN the two Lifshitz-Hölder exponents correspond and present a possible explanation for the origin of the bifractality. Finally, we extend our deterministic model of FSFNs to a stochastic one which includes multiple generators and explore the possibility that FSFNs formed by the extended model may come to exhibit the multifractality.

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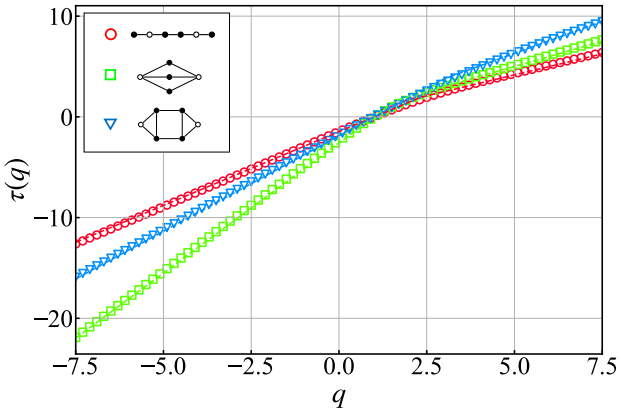

Figure 1 q-dependence of the mass exponents