Bifractal Property of Stochastic Scale-free Networks

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Various real-world networks are characterized by heterogeneous distributions of degrees (scale-free property) and power-law relations between the number of nodes and the network diameter (fractal property). When scale-free and fractal properties coexist in a network, the network structure is known to be multifractal [1]. Some fractal scale-free networks (FSFNs) can be hierarchically generated by a deterministic model. In this model [2], the t-th generation network is formed by replacing iteratively every edge of the $(t-1)$-th generation network with a small graph called a generator. In the previous meeting, we presented that FSFNs formed by this model display bifractal structures that are described by two local fractalities. It is, however, not clear how general the bifractal nature of FSFNs is.

In this work, we first propose a stochastic model of hierarchical FSFNs in which every edge of the previous generation network is replaced by one of multiple generators with a certain probability. This model encompasses a much broader class of FSFNs than the deterministic one. We then perform the multifractal analysis of FSFNs formed by this stochastic model. The results indicate that FSFNs by this model are also bifractal. In addition, we elucidate the bifractality of fractal scale-free networks, that are not formed by a hierarchical scheme. These results suggest that any fractal scale-free network takes a bifractal structure.

[1] S. Furuya and K. Yakubo, Physical Review E84, 036118 (2011).
[2] K. Yakubo and Y. Fujiki, arXiv:2109.00703 (2021).